Try 22 people:
P(22 people at least two have same birthday)
$= 1 - P$(no one)
$= 1 - \dfrac{(365)(364)\ldots(344)}{(365)^{22}}$
$\doteq 0.4757$
The probability that in a group of 22 people two will have the same birthday is less than 0.5. The group needs to consist of 23 people for the probability to be more than 0.5.

17. Answers may vary. Use the simulation methods discussed in the text to simulate 30 sets of number representing months and days. Use the result to find out number of people who have birthday in June.

18. Answers may vary. Use the simulation methods discussed in the text to simulate 12 numbers representing whether the milk has expired or not. Be sure 75% of the milk is sold before it expires. Use the result to find out the probability of throwing three or more of these cartoons.

19. Since the newspaper only 50 polled people, its result of 60% in favour of the new expressway could be different from the actual percent of the popuation in favour.
Given that the probability of voting *yes* or *no* is 0.5,
Probability of 30 *yes* votes
$= \binom{50}{30}(0.5)^{30}(0.5)^{50-30} \doteq 0.0419$

20. Answers may vary.

Chapter 5 Wrap-Up, page 324

1.(a) Let the random variable X be the amount of winnings in dollars.

Outcome (X)	0	2	5
$P(X)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

(b) $E(X) = 0\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right) + 5\left(\frac{1}{4}\right) = \frac{7}{4} = 1.75$
The expected winning of each game is \$1.75.

(c) No, since the probability of HH and TT are the same and their winnings are different. Also, the expectation of winning should be 2 since it cost 2 dollars per game.

2.(a) Let X be the number of students on the committee.
$P(X) = \dfrac{\binom{5}{X}\binom{4}{4-X}}{\binom{9}{4}}$ $\qquad P(4) = \dfrac{\binom{5}{4}\binom{4}{0}}{\binom{9}{4}}$
$= \dfrac{5}{126}$
$\doteq 0.04$
$1 - 0.04 = 0.96$
The probability that there is a teacher on the committee is 0.96.

(b) The expected number of students
$= np = 4\left(\frac{5}{9}\right) \doteq 2.22$
Note: distribution of choosing the committee is hyper-geometric distribution and expectation of hyper-geometric distribution is $n\left(\frac{r}{N}\right)$, where n is the number of people being chosen for the committee, r is the number of students, and N is the size of the population.

3.(a) C(4 + 7, 4) = C(11, 4) = 330
There are 330 routes.

(b) C(1 + 7, 1) = C(8, 1) = 8
There are 8 routes.

4.(a) $(3x + 4y)^5$
$= \binom{5}{0}(3x)^5(4y)^0 + \binom{5}{1}(3x)^4(4y)^1 + \binom{5}{2}(3x)^3(4y)^2$
$+ \binom{5}{3}(3x)^2(4y)^3 + \binom{5}{4}(3x)^1(4y)^4 + \binom{5}{5}(3x)^0(4y)^5$
$= (1)(243x^5)(1) + (5)(81x^4)(4y) + (10)(27x^3)(16y^2)$
$+ (10)(9x^2)(64y^3) + (5)(3x)(256y^4) + (1)(1)(1024y^5)$
$= 243x^5 + 1620x^4y + 4320x^3y^2 + 5760x^2y^3$
$+ 3840xy^4 + 1024y^5$

(b) $(2x^3 - 3x^2)^6$
$= \binom{6}{0}(2x^3)^6(-3x^2)^0 + \binom{6}{1}(2x^3)^5(-3x^2)^1$
$+ \binom{6}{2}(2x^3)^4(-3x^2)^2 + \binom{6}{3}(2x^3)^3(-3x^2)^3$
$+ \binom{6}{4}(2x^3)^2(-3x^2)^4 + \binom{6}{5}(2x^3)^1(-3x^2)^5$
$+ \binom{6}{6}(2x^3)^0(-3x^2)^6$
$= (1)(64x^{18})(1) + (6)(32x^{15})(-3x^2) + (15)(16x^{12})(9x^4)$
$+ (20)(8x^9)(-27x^6) + (15)(4x^6)(81x^8)$
$+ (6)(2x^3)(-243x^{10}) + (1)(1)(729x^{12})$
$= 64x^{18} - 576x^{17} + 2160x^{16} - 4320x^{15} + 4860x^{14}$
$- 2916x^{13} + 729x^{12}$

(c) $\left(5-\frac{2}{x}\right)^3 = (5-2x^{-1})^3$

$= \binom{3}{0}(5)^3(-2x^{-1})^0 + \binom{3}{1}(5)^2(-2x^{-1})^1$

$+ \binom{3}{2}(5)^1(-2x^{-1})^2 + \binom{3}{3}(5)^0(-2x^{-1})^3$

$= (1)(125)(1) + (3)(25)(-2x^{-1})$

$+ (3)(5)(4x^{-2}) + (1)(1)(-8x^{-3})$

$= 125 - 150x^{-1} + 60x^{-2} - 8x^{-3}$

$= 25 - \frac{150}{x} + \frac{60}{x^2} - \frac{8}{x^3}$

(d) $\left(x+\sqrt{x}\right)^4 = (x + x^{0.5})^4$

$= \binom{4}{0}(x)^4(x^{0.5})^0 + \binom{4}{1}(x)^3(x^{0.5})^1 + \binom{4}{2}(x)^2(x^{0.5})^2$

$+ \binom{4}{3}(x)^1(x^{0.5})^3 + \binom{4}{4}(x)^0(x^{0.5})^4$

$= (1)(x^4)(1) + (4)(x^3)(x^{0.5}) + (6)(x^2)(x^1)$

$+ (4)(x)(x^{1.5}) + (1)(1)(x^2)$

$= x^4 + 4x^{3.5} + 6x^3 + 4x^{2.5} + x^2$

5.(a) There are 8 + 1 = 9 terms.

(b) $t_{r+1} = \binom{8}{r}\left(\frac{3}{x}\right)^{8-r}\left(-x^3\right)^r = (-1)^r\binom{8}{r}(3)^{8-r}\left(\frac{x^{3r}}{x^{8-r}}\right)$

$\frac{x^{3r}}{x^{8-r}} = 1$

$x^{3r} = x^{8-r}$

$3r = 8 - r$

$4r = 8$

$r = 2$

$t_{2+1} = (-1)^2\binom{8}{2}(3)^6\left(\frac{x^6}{x^6}\right) = (1)(28)(729)(1)$

$t_3 = 20\ 412$

(c) $\frac{x^{3r}}{x^{8-r}} = x^{12}$

$x^{3r} = x^{12+8-r} = x^{20-r}$

$3r = 20 - r$

$4r = 20$

$r = 5$

$t_{5+1} = (-1)^5\binom{8}{5}(3)^3\left(\frac{x^{15}}{x^3}\right) = (-1)(56)(27)(x^{12})$

$t_6 = -1512x^{12}$

6.(a)

$$\frac{\binom{74}{21}}{\binom{73}{20}} = \frac{\frac{74!}{21!53!}}{\frac{73!}{20!53!}} = \frac{74!20!53!}{73!21!53!} = \frac{(74)73!20!53!}{(21)73!20!53!} = \frac{74}{21}$$

(b)

$\binom{20}{15} + \binom{20}{16} = \binom{21}{16}$ by Pascal's Identity

$\binom{21}{16} = 20\ 349$

7.(a) Let X be the number of heads.

$P(X) = \binom{10}{X}(0.5)^X(0.5)^{10-X} = \binom{10}{X}(0.5)^{10}$

$P(5) = \binom{10}{5}(0.5)^{10} \doteq 0.246$

(b) $P(8) = \binom{10}{8}(0.5)^{10} \doteq 0.044$

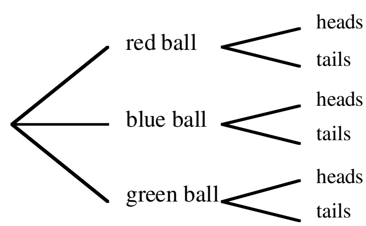
(c) $P(2) = \binom{10}{2}(0.5)^{10} \doteq 0.044$

(d) $P(x \geq 2) = 1 - P(0) - P(1)$

$= 1 - \binom{10}{0}(0.5)^{10} - \binom{10}{1}(0.5)^{10}$

$\doteq 0.989$

8.(a)



(b) Answers may vary.

(c) $P(\text{keeping a red ball}) = \frac{3}{3+1+1} \times \frac{1}{2} = \frac{3}{10}$ or 0.3

(d) No, since the probability may change after each draw.

9. Let X be the number of sales.

$P(X) = \binom{4}{X}\left(\frac{1}{3}\right)^X\left(\frac{2}{3}\right)^{4-X}$

$P(X \geq 2) = 1 - P(0) - P(1)$

$= 1 - \binom{4}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^4 - \binom{4}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3$

$\doteq 0.407$

The probability that she will have at least 2 sales is 0.407.

10.(a) Let x be the number of correct guesses.

$P(X) = \binom{10}{X}(0.25)^X(0.75)^{10-X}$

$P(5) = \binom{10}{5}(0.25)^5(0.75)^5 \doteq 0.058$

(b) $P(6) = \binom{10}{6}(0.25)^6(0.75)^4 \doteq 0.016$

(c) $P(1) = \binom{10}{1}(0.25)^1(0.75)^9 \doteq 0.188$

(d) $P(10) = \binom{10}{10}(0.25)^{10}(0.75)^0 \doteq 0.000\,001$

11.(a) The probability of getting a double 3 is $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$.

Let X be the number of double 3s obtain.

$P(X) = \binom{30}{X}\left(\frac{1}{36}\right)^X\left(\frac{35}{36}\right)^{30-X}$

$P(5) = \binom{30}{5}\left(\frac{1}{36}\right)^5\left(\frac{35}{36}\right)^{25} \doteq 0.001\,165$

(b) $P(X \geq 1) = 1 - P(0)$

$= 1 - \binom{30}{0}\left(\frac{1}{36}\right)^0\left(\frac{35}{36}\right)^{30} \doteq 0.570\,497$

12.(a) Let X be the number of defective pads.

$P(X) = \binom{200}{X}(0.002)^X(0.998)^{200-X}$

$P(X \geq 1) = 1 - P(0)$

$= 1 - \binom{200}{0}(0.002)^0(0.998)^{200} \doteq 0.330$

(b) $E(X) = np = (200)(0.002) = 0.4$

13.(a) There are C(16, 4) = 1820 to choose 4 marbles

There are C(10, 3) × C(6, 1) = 720 to choose 3 white marbles and 1 red marble.

$P(\text{3 white marbles}) = \frac{720}{1820} = 0.396$

(b) No, because the probabilities change every time a marble is drawn.

(c) P(0 white marbles)

$= \frac{C(10,0)\times C(6,4)}{C(16,4)} = \frac{15}{1820} \doteq 0.008$

P(1 white marble)

$= \frac{C(10,1)\times C(6,3)}{C(16,4)} = \frac{200}{1820} \doteq 0.110$

P(2 white marbles)

$= \frac{C(10,2)\times C(6,2)}{C(16,4)} = \frac{675}{1820} \doteq 0.371$

P(3 white marbles)

$= \frac{C(10,3)\times C(6,1)}{C(16,4)} = \frac{720}{1820} \doteq 0.396$

P(4 white marbles)

$= \frac{C(10,4)\times C(6,0)}{C(16,4)} = \frac{210}{1820} \doteq 0.115$

X	0	1	2	3	4
$P(X)$	0.008	0.110	0.371	0.396	0.115

(d) Answers may vary.

(e) $E(X) = 0(0.008) + 1(0.110) + 2(0.371) + 3(0.396) + 4(0.115) = 2.5$

The number of expected white marbles is 2.5.

14.(a) P(5 showing 6 out of 10 times)

$= \binom{10}{6}\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^4 = 0.002\,171$

The probability of this occurring is 0.002.

(b) Answers may vary but the expected number of times 5 occur is $\frac{10}{6} = 1.7$.

Chapter 5 Test, page 326

1.(a) Let X be defined as the random variable that counts the transaction time (in minutes) with tellers during a 2–hour period.

(b) Total number of transactions

= 20 + 12 + 9 + 5 + 3 + 1 = 50

Time (min)	1	2	3
Frequency	$\frac{20}{50} = 0.40$	$\frac{12}{50} = 0.24$	$\frac{9}{50} = 0.18$
Time (min)	4	5	6 or more
Frequency	$\frac{5}{50} = 0.10$	$\frac{3}{50} = 0.06$	$\frac{1}{50} = 0.02$

(c) $E(X) = 1(0.40) + 2(0.24) + 3(0.18) + 4(0.10) + 5(0.06) + 6(0.02) = 2.24$

The expected transaction time is 2.24 minutes.

2.(a) $p = 0.275$, $n = 4$

Let X be the number of hits the player gets.

$P(X) = \binom{4}{X}(0.275)^X(0.725)^{4-X}$

$P(3) = \binom{4}{3}(0.275)^3(0.725)^1 \doteq 0.060$

(b) P(at least 1 hit) = 1 – $P(0)$

$= 1 - \binom{4}{0}(0.275)^0(0.725)^4 \doteq 0.724$

(c) $E(X) = np = (4)(0.275) = 1.100$

The expected number of hits is 1.

The probability of getting exactly one hit is

$P(1) = \binom{4}{1}(0.275)^1(0.725)^{4-1} \doteq 0.419$

3.(a) Answers may vary.

(b) Let X be the number of defective processors.

$P(X) = \binom{12}{X}(0.1)^X(0.9)^{12-X}$

Shipment accepted if only 0 or 1 defective found.

P(shipment accepted) = $P(0) + P(1)$

$= \binom{12}{0}(0.1)^0(0.9)^{12} + \binom{12}{1}(0.1)^1(0.9)^{11} \doteq 0.659$

The probability of being accepted is 0.659.